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Résumé

Cet article présente un modèle mathématique déterministe et un modèle stochastique afin de maximiser les profits découlant de la vente d'électricité produite par un réseau de barrages hydroélectriques. Le premier modèle suppose que les prix de l'énergie sont déterministes tandis que le deuxième intègre des éléments stochastiques en gérant un arbre de scénarios potentiels pour les prix. Des résultats numériques basés sur des données historiques démontrent la supériorité du modèle stochastique sur le modèle déterministe. Il est également démontré que la volatilité des prix a un impact significatif sur les profits additionnels générés par le modèle stochastique.

Mots-clés : hydroélectricité, marché de l'énergie, prix, barrages, réservoirs, centrales, programmation mathématique, programmation stochastique.

Abstract

This paper presents a deterministic and a stochastic mathematical model for maximizing the profits earned by selling electricity produced through a cascade of dams and reservoirs. The first model is based on deterministic energy prices while the other integrates stochastic through the management of a tree of potential price scenarios. Numerical results based on historical data demonstrate the superiority of the stochastic model over the deterministic one. It is also shown that price volatility significantly impacts the additional profits obtained by the stochastic model.

Keywords: hydroelectricity, energy market, prices, dams, reservoirs, power plants, mathematical programming, stochastic programming.

1 Introduction

As more and more countries deregulate their electricity market, new challenges appear for hydroelectricity producers. They now need to consider the selling price of electricity when they design their production plan to maximize their profits. This is to be opposed to maximizing the total production when prices are constant.

The problem under study thus involves both hydrological and energy market issues. Hydrological issues are related to the management of hydroelectric facilities, in particular the management of water that must be released at each site. Energy market issues are concerned with the setting of prices for the electricity produced and the reserve, based on demands from buyers and offers from producers. Since the bidding process introduces an additional level of complexity (see below), it is not explicitly addressed here. Rather, we assume that the electricity produced and the reserve are sold at the given, deterministic or stochastic, prices.

The original contribution of this article is thus to model a complete hydrological system that involves a series of facilities along a river and to derive a production plan in this setting that accounts for stochastic energy market prices.

1.1 Hydrological model

There are four important sites on the river under study. The first site, P_0, is a dam that retains water from a big reservoir just behind it. The three other sites, P_1, P_2 and P_3, contain an hydroelectric power plant, a dam and a reservoir. Each power plant produces electricity using many turbines that are run independently. The sites are connected in sequence, namely (P_0, P_1, P_2, P_3). Hence, the water released at the upstream dam of site P_i goes into the reservoir of the downstream site P_{i+1}. This simple organization is illustrated in Figure 1. In this figure, a triangle stands for a reservoir with a dam while a little square represents a power plant.

The time taken by the water flow to go from one site to the next is referred to as the "river routing effect". This complex phenomenon can be approximated through river routing coefficients. The latter corresponds to the fraction of water released upstream that arrives at the downstream site every hour after the release. Natural inflows must also be taken into account, like those coming from snow melting, rain, runoff water and natural river flow (for the head reservoir). These inflows are stochastic, but are handled here in a deterministic way, by using mean hourly values provided by a natural inflow forecast model.

To avoid structural damage, the water level at each dam must also lie between a minimum and a maximum value, which are the same all year round. Each dam possesses a mechanism to spill a large quantity of water, if necessary. The spill flow is controlled quite precisely by adjusting gate openings. However, spilling should be avoided as much as possible, given that no electricity is produced in this case.

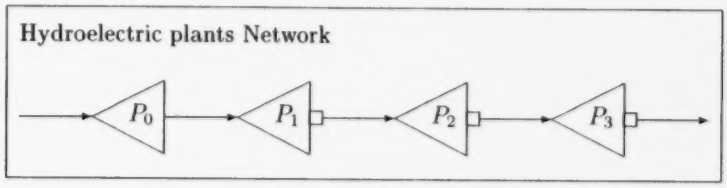

Figure 1: Hydroelectric Plants Network

The amount of electricity produced by each turbine (unit) is a function of the water head, unit flow and unit type. These curves are determined empirically and have, in general, a polynomial form. When a unit is started, it suffers some wear because of the huge water pressure applied to it. So, there is a cost associated with starting a unit, and this cost is also taken into account. Besides producing electricity, a unit can also be in "reserve", which means that some of its power is put aside to provide electricity, in case of a shortcoming

somewhere over the network. A revenue is earned through this practice depending on the type of reserve under consideration, which can be either "10 minutes spin", "10 minutes non spin" or "30 minutes non spin". It should be noted that when all the power of a unit is assigned to the "10 minutes spin" reserve, a cost is incurred because this unit needs some electricity to spin at the right speed to synchronize itself with the electric network.

1.2 Electricity market model

In our application, the electricity prices come from a deregulated market, where producers and buyers submit selling and purchase bids, respectively. Every bid corresponds to some quantity of energy and a price per MWh. Supply and demand curves are then created and the intersection of the two curves determines the Market Clearing Price (MCP). All selling bids under the MCP and all purchase bids over the MCP are accepted. The producer with at least one accepted bid will receive the MCP for each MWh that he has to produce. The buyer with at least one accepted bid will pay the MCP for each MWh that he will use.

In addition to this energy market, there are three other markets associated with the three reserve types. However, the "30 minutes non spin" reserve is not considered here because its price is always lower than or equal to the "10 minutes non spin" reserve price, and there is no cost for providing these two reserves. The price for each reserve type is calculated by considering bids from producers and the amount of reserve required for the electric network to be safe.

As mentioned previously, bidding issues are not explicitly addressed in this work. We directly use historical market price data to feed our models (averages for the deterministic model; distributions that follow the evolution of prices over the day for the stochastic model).

1.3 Optimization

The goal is to find a production plan, for each unit in the power plants along the river, that maximizes the profit resulting from energy and reserve selling, while satisfying specific operational and physical constraints. The result is a table indicating, for each hour and unit, how the power of each turbine is distributed between electricity production and reserve over a 24-hour horizon.

To avoid emptying reservoirs, the profit is modified through the use of target volumes of water. That is, when the volume of water in a reservoir is above the target at the end of the planning horizon, a reward or bonus is added according to the "value" of this water (based on long term price averages). Conversely, a penalty is incurred if the final volume is below the target.

2 Literature review

Besides papers on optimization of hydroelectric production, two related classes of problems are also widely reported in the literature, namely, reservoir management and hydrothermal plant coordination problems. The latter are complex problems that share many similarities with our problem. They are thus also briefly reviewed in the following.

2.1 Reservoir management problems

These problems are concerned with the management of water levels in a network of reservoirs. The objective varies widely from one application to another and there are also different operational constraints that must be satisfied. The water in these reservoirs can be used for multiple purposes, like hydroelectricity generation, water consumption or irrigation, etc. [5]. This is a rather mature field of research with plenty of literature, particularly on flood control. A state-of-the-art review can be found in [53].

These problems become very complex when a sequence of reservoirs must be managed. Indeed, any water released at an upstream reservoir is part of the inflows at the downstream reservoir. The timing of the water

releases thus becomes an important issue, due to the lag time between the upstream release and the water arrival at the downstream reservoir.

Another important issue is the stochastic nature of the natural inflows. To address this issue, stochastic dynamic programming techniques look as a "natural" problem-solving approach. Furthermore, it allows non-convexities to be taken into account (due, for example, to head effects). Unfortunately, for realistic problems with many reservoirs, the computational burden is often excessive. Different aggregation techniques to reduce problem size have thus been proposed in the literature [4, 43, 50]. Other methods like stochastic programming [42, 52], fuzzy rules [20], neural networks [21] and ant colony systems [26] have also been successfully applied to this kind of problem.

2.2 Hydrothermal coordination problems

The goal here is to improve the coordination between hydroelectric and thermal plants. This can be very beneficial, as hydroelectric plants can be operated in a quite flexible way, but are limited by the amount of water available. On the other hand, thermal plants are less flexible but produce electricity more steadily. The main idea is thus to use hydroelectric facilities during peak hours to supplement thermal plants and to use the thermal plants otherwise. Fuel costs and efficiency curves of thermal plants can also be taken into account during the optimization process.

A variety of optimization techniques have been proposed to solve this problem. In the deterministic case, problem-solving approaches based on mathematical programming [46, 54], neural networks [33, 35], specialized heuristics [47], Lagrangian relaxation [18, 31, 36, 37] and metaheuristics, like genetic algorithms [7, 14, 23, 38] and ant colony systems [24] are reported in the literature.

When stochastic issues are considered, three main optimization techniques are reported, namely, fuzzy rules [32], stochastic programming [10, 16] and stochastic dynamic programming [9, 15, 27, 41].

2.3 Hydroelectricity production problems

In this class of problems, the main objective deals with hydroelectric generation. This class is divided into two subclasses, depending if there is only one reservoir or many reservoirs. The latter are much more difficult to solve because of the "cascade" effects from one reservoir to another.

2.3.1 Single reservoir

Although one-reservoir problems are easier to solve than multiple-reservoir problems, they can be quite challenging if stochastic natural inflows are considered. Other issues can also be considered like head effects or turbine start-up costs.

In [39], the authors describe a problem-solving approach based on dynamic programming, while specialized heuristics are reported in [30, 51]. In [29], a version with stochastic inflows is addressed through stochastic dynamic programming. Single reservoir problems were firsts studied in the literature, but the focus has since shifted to multiple reservoir problems.

2.3.2 Multiple reservoirs

One way to maximize water utilization is to build a series of hydroelectric facilities along a river. Accordingly, most rivers with good hydroelectric potential have many plants organized "in cascade" on their shore. Table 1 classifies the papers dealing with these problems along two dimensions: electricity prices and natural inflows.

In this table, "no price" means that electricity prices are not considered. In most cases, it means that electricity production is maximized instead of the revenue generated from selling electricity. We did not find any paper dealing with the optimization of bids in a deregulated electricity market. Integrating a market makes the problem much more complex, because the active periods where electricity should be produced cannot be known with certainty. This uncertainty must be taken into account when managing the reservoir levels.

Price type	Natural inflows type	Papers
no price	deterministic	[2, 22, 40, 44, 45, 48, 55]
no price	stochastic	[1, 3, 6, 11]
deterministic	deterministic	[8, 13, 17, 19, 49]
deterministic	stochastic	
stochastic	deterministic	
stochastic	stochastic	[12]

Table 1: Papers on hydroelectricity production with multiple reservoirs (by price and natural inflow types)

We have classified the papers by price and natural inflow types, because the latter are good indicators of the problem difficulty. Problems with stochastic prices and natural inflows are particularly difficult to solve (a good review of different optimization techniques used in stochastic contexts can be found in [28]).

Table 2 classifies papers on multiple reservoir problems based on the proposed methodology. As we can see, many different optimization techniques have been reported. All these methods deal with deterministic problems, except for stochastic dynamic programming. It should also be noted that, at the best of our knowledge, stochastic programming has never been applied to these problems.

Problem-solving methods	Papers
Network programming	[2, 12, 19]
Linear programming	[40, 48, 49]
Successive linear programming	[12, 17, 44]
Mixed integer programming	[8, 13, 48, 49]
Dynamic programming	[19]
Stochastic dynamic programming	[1, 3, 6, 11, 12]
Genetic algorithm	[55]
Neural network	[45]
Heuristics	[22]
Nested benders decomposition	[3]
Lagrangian relaxation	[19, 45]
Branch and bound	[2]

Table 2: Papers on hydroelectric production with multiple reservoirs (by methodology)

3 Mathematical Models

In the following, we formally introduce two variants of the problem introduced in Section 1. In the first case, a model where the electricity prices are assumed to be deterministic is presented. Then, a second model that accounts for the stochastic nature of these prices is described.

3.1 Deterministic model

Here, the electricity and reserve prices are assumed to be known over all periods of the production horizon.

3.1.1 Parameters

- P : number of periods for production planning.

- D : number of dams.
- U_d : number of units in dam d, $d = 1, .., D$.
- DUR_p : duration of period p in hours, $p = 1, .., P$.
- P_p^{ENGY} : price for each MWh of electricity produced in period p (\$/MWh), $p = 1, .., P$.
- P_p^{10S} : price for each MWh of electricity offered in "10 minute spin reserve" in period p (\$/MWh), $p = 1, .., P$.
- P_p^{10N} : price for each MWh of electricity offered in "10 minute non spin reserve" in period p (\$/MWh), $p = 1, .., P$.
- $PROD_{d,u}(f)$: power of unit u of dam d as a function of flow f (MW), $d = 1, .., D$; $u = 1, .., U_d$ (see section 3.1.7).
- $I_{d,u}$: number of piecewise parts in the linear approximation of $PROD_{d,u}(f)$, $d = 1, .., D$; $u = 1, .., U_d$. (see section 3.1.7).
- $RATE_{d,u,i}$: rate (slope) of part i in the linear approximation of $PROD_{d,u}(f)$, $d = 1, .., D$; $u = 1, .., U_d$; $i = 1, .., I_{d,u}$ (see section 3.1.7).
- $LIM_{d,u,i}$: upper bound or limit of part i in the linear approximation of $PROD_{d,u}(f)$, $d = 1, .., D$; $u = 1, .., U_d$; $i = 1, .., I_{d,u} - 1$ (see section 3.1.7).
- $VOL_{d,p}^{min}$: minimum volume of dam d at the end of period p (m^3), $d = 1, .., D$; $p = 1, .., P$.
- $VOL_{d,p}^{max}$: maximum volume of dam d at the end of period p (m^3), $d = 1, .., D$; $p = 1, .., P$.
- VOL_d^{init} : initial volume of dam d (at the end of period 0) (m^3), $d = 1, .., D$.
- VOL_d^{final} : target volume of dam d at the end of the last period P (m^3/s), $d = 1, .., D$.
- $F_{d,u}^{min}$: minimum amount of flow that can be processed by unit u of dam d (m^3/s), $d = 1, .., D$; $u = 1, .., U_d$.
- $F_{d,u}^{max}$: maximum amount of flow that can be processed by unit u of dam d (m^3/s), $d = 1, .., D$; $u = 1, .., U_d$.
- $F_d^{spill_{min}}$: minimum spill flow of dam d (m^3/s), $d = 1, .., D$.
- $F_d^{spill_{max}}$: maximum spill flow of dam d (m^3/s), $d = 1, .., D$.
- $F_d^{out_{min}}$: minimum outflow of dam d (m^3/s), $d = 1, .., D$.
- $F_d^{out_{max}}$: maximum outflow of dam d (m^3/s), $d = 1, .., D$.
- $MW_{d,u}^{max}$: maximum power of unit u of dam d (MW), $d = 1, .., D$; $u = 1, .., U_d$.
- NAT_IFd, p : mean natural inflow forecast at dam d for period p (m^3/s), $d = 1, .., D$; $p = 1, .., P$.
- $LK_{z,d}$: 1 if dam z is the immediate predecessor of dam d, 0 otherwise, $z = 1, .., D$; $d = 1, .., D$.
- N : number of periods that are considered when calculating the river routing effect.
- $LK_FRAC_{z,d,t}$: fraction ($\in [0, 1]$) of water, released in period $p - t$ at dam z, that arrives at dam d in period p (same value for all p), $z = 1, .., D$; $d = 1, .., D$; $t = 0, .., N$.
- $MW_{d,u}^{spin}$: power needed by unit u of dam d to make it spin (MW), $d = 1, .., D$; $u = 1, .., U_d$.
- $C_Start_{d,u}$: cost for each start up of unit u of dam d (\$), $d = 1, .., D$; $u = 1, .., U_d$.
- E_d : estimate of the energy that can be produced for each m^3 of water at dam d (MWh/m^3), $d = 1, .., D$. This value gets larger as we move upstream (upward) because more electricity can be produced with the same amount of water, due to the cascade effect.
- LT^{ENGY} : estimate of the average long term electricity price (\$/MWh), $d = 1, .., D$.

3.1.2 Decision variables

- $f_{d,p}^{spill} \geq 0$: flow spilled at dam d in period p (m^3/s), $d = 1, .., D$; $p = 1, .., P$.
- $f_{d,u,p}^{ENGY} \geq 0$: flow going through unit u of dam d in period p to produce electricity (m^3/s), $d = 1, .., D$; $u = 1, .., U_d$; $p = 1, .., P$.

- $f^{ENGY}_{d,u,p,i} \geq 0$: contribution of piecewise part i, in the linear approximation of $PROD_{d,u}(f)$, to the flow going through unit u of dam d in period p to produce electricity (m^3/s), $d = 1,..,D$; $u = 1,..,U_d$; $p = 1,..,P$; $i = 1,..,I_{d,u}$. (see section 3.1.7).
- $b^{ENGY}_{d,u,p} \in \{0,1\}$: 1 if electricity is produced by unit u of dam d in period p, 0 otherwise, $d = 1,..,D$; $u = 1,..,U_d$; $p = 0,1,..,P$.
- $b^{spin}_{d,u,p} \in \{0,1\}$: 1 if unit u of dam d spins by supplying electricity to it in period p, 0 otherwise, $d = 1,..,D$; $u = 1,..,U_d$; $p = 1,..,P$.
- $mw^{ENGY}_{d,u,p} \geq 0$: power used to produce electricity for unit u of dam d in period p (MW), $d = 1,..,D$; $u = 1,..,U_d$; $p = 1,..,P$.
- $mw^{10S}_{d,u,p} \geq 0$: power reserved for "10 minutes spin reserve" for unit u of dam d in period p (MW), $d = 1,..,D$; $u = 1,..,U_d$; $p = 1,..,P$.
- $mw^{10N}_{d,u,p} \geq 0$: power reserved for "10 minutes non spin reserve" for unit u of dam d in period p (MW), $d = 1,..,D$; $u = 1,..,U_d$; $p = 1,..,P$.

3.1.3 Constraints

- (C1) $f^{ENGY}_{d,u,p}$ is the sum of all piecewise parts, $d = 1,..,D$; $u = 1,..,U_d$; $p = 1,..,P$.

$$f^{ENGY}_{d,u,p} = \sum_{i=1,..,I_{d,u}} f^{ENGY}_{d,u,p,i}$$

- (C2) Bounds on $f^{ENGY}_{d,u,p}$, $d = 1,..,D$; $u = 1,..,U_d$; $p = 1,..,P$.

$$0 \leq f^{ENGY}_{d,u,p} \leq F^{max}_{d,u}$$

- (C3) Bounds on $f^{ENGY}_{d,u,p,i}$, $d = 1,..,D$; $u = 1,..,U_d$; $p = 1,..,P$; $i = 1,..,I_{d,u}$.

$$\begin{aligned} i = 1: &\quad 0 \leq f^{ENGY}_{d,u,p,i} \leq LIM_{d,u,1} \\ i = 2,..I_{d,u}-1: &\quad 0 \leq f^{ENGY}_{d,u,p,i} \leq LIM_{d,u,i+1} - LIM_{d,u,i} \\ i = I_{d,u}: &\quad 0 \leq f^{ENGY}_{d,u,p,i} \leq F^{max}_{d,u} \end{aligned}$$

- (C4) Calculation of $mw^{ENGY}_{d,u,p} = PROD_{d,u}(f^{ENGY}_{d,u,p})$ through the linear approximation, $d = 1,..,D$; $u = 1,..,U_d$; $p = 1,..,P$.

$$mw^{ENGY}_{d,u,p} = \sum_{i=1,..,I_{d,u}} (f^{ENGY}_{d,u,p,i} * RATE_{d,u,i}) - b^{ENGY}_{d,u,p} * F^{min}_{d,u} * RATE_{d,u,1}$$

- (C5) $f^{ENGY}_{d,u,p}$ equals 0 if $b^{ENGY}_{d,u,p}$ equals 0 and is between a minimum and a maximum value otherwise, $d = 1,..,D$; $u = 1,..,U_d$; $p = 1,..,P$.

$$F^{min}_{d,u} \cdot b^{ENGY}_{d,u,p} \leq f^{ENGY}_{d,u,p} \leq F^{max}_{d,u} \cdot b^{ENGY}_{d,u,p}$$

- (C6) Spill flow of dam d in period p is between a minimum and a maximum value, $d = 1,..,D$; $p = 1,..,P$.

$$F^{spill_{min}}_{d} \leq f^{spill}_{d,p} \leq F^{spill_{max}}_{d}$$

- (C7) Maximum power of unit u of dam d is used at period p, $d = 1,..,D$; $u = 1,..,U_d$; $p = 1,..,P$.

$$mw^{ENGY}_{d,u,p} + mw^{10S}_{d,u,p} + mw^{10N}_{d,u,p} = MW^{max}_{d,u}$$

- (C8) Power for "10 minutes spin reserve" can only be reserved if unit u of dam d spins or produces electricity, $d = 1,..,D$; $u = 1,..,U_d$; $p = 1,..,P$.

$$mw^{10S}_{d,u,p} \leq MW^{max}_{d,u} \cdot (b^{ENGY}_{d,u,p} + b^{spin}_{d,u,p})$$

- (C9) Unit u of dam d cannot produce electricity and spin at the same time, $d = 1,..,D$; $u = 1,..,U_d$; $p = 1,..,P$.

$$b^{ENGY}_{d,u,p} + b^{spin}_{d,u,p} \leq 1$$

3.1.4 Intermediate variables

Here is a list of intermediate variables. The values of these variables depend on the parameters and the original decision variables. They are used here to simplify the formulation of the constraints that follow.

- $inflow_{d,p}$: flow that enters the reservoir of dam d in period p (m^3/s), $d = 1, .., D$; $p = 1, .., P$.

$$inflow_{d,p} = NAT_IF_{d,p} + \sum_{\substack{z=1,..,D \\ t=0,..,N}} LK_{z,d} * outflow_{z,p-t} * LK_FRAC_{z,d,t}$$

- $outflow_{d,p}$: flow that leaves the reservoir of dam d in period p (m^3/s), $d = 1, .., D$; $p = 1, .., P$. It is also assumed that a number of "historical" outflow values are available for periods $-(N-1), .., 0$.

$$outflow_{d,p} = f^{spill}_{d,p} + \sum_{u=1,..,U_d} f^{ENGY}_{d,u,p}$$

- $vol_{d,p}$: water volume of dam d at the end of period p (m^3), $d = 1, .., D$; $p = 1, .., P$.

$$\begin{aligned} p = 1: &\quad vol_{d,p} = VOL^{init}_d + (inflow_{d,p} - outflow_{d,p}) * DUR_p * 3600 \\ p = 2, .., P: &\quad vol_{d,p} = vol_{d,p-1} + (inflow_{d,p} - outflow_{d,p}) * DUR_p * 3600 \end{aligned}$$

3.1.5 Constraints on intermediate variables

- (C10) Water volume of dam d at the end of period p is between a minimum and a maximum value, $d = 1, .., D$; $p = 1, .., P$.

$$VOL^{min}_{d,p} \leq vol_{d,p} \leq VOL^{max}_{d,p}$$

- (C11) Outflow of dam d in period p is between a minimum and a maximum value, $d = 1, .., D$; $p = 1, .., P$.

$$F^{out_{min}}_d \leq outflow_{d,p} \leq F^{out_{max}}_d$$

3.1.6 Objective

- Objective function ($)

$$Maximize \sum_{\substack{d \in \{1,..,D\} \\ u \in \{1,..,U_d\} \\ p \in \{1,..,P\}}} \{revenue_{d,u,p} - c^{spin}_{d,u,p} - c^{start}_{d,u,p}\} + \sum_{d \in \{1,..,D\}} volAdj_d$$

- $revenue_{d,u,p}$: revenue from selling electricity and reserve for unit u of dam d in period p ($), $d = 1, .., D$; $u = 1, .., U_d$; $p = 1, .., P$.

$$revenue_{d,u,p} = (mw^{ENGY}_{d,u,p} \cdot P^{ENGY}_p + mw^{10S}_{d,u,p} \cdot P^{10S}_p + mw^{10N}_{d,u,p} \cdot P^{10N}_p) \cdot DUR_p$$

- $c^{spin}_{d,u,p}$: cost of providing electricity to unit u of dam d to make it spin in period p ($), $d = 1, .., D$; $u = 1, .., U_d$; $p = 1, .., P$.

$$c^{spin}_{d,u,p} = b^{spin}_{d,u,p} \cdot MW^{spin}_{d,u} \cdot P^{ENGY}_p \cdot DUR_p$$

- $c^{start}_{d,u,p}$: starting cost of unit u of dam d at the start of period p ($), $d = 1, .., D$; $u = 1, .., U_d$; $p = 2, .., P$.

$$\begin{aligned} p = 1: &\quad c^{start}_{d,u,p} \geq C_START_{d,u} \cdot b^{ENGY}_{d,u,p}, \\ p = 2, .., P: &\quad c^{start}_{d,u,p} \geq C_START_{d,u} \cdot (b^{ENGY}_{d,u,p} - b^{ENGY}_{d,u,p-1}), \\ &\quad c^{start}_{d,u,p} \geq 0 \end{aligned}$$

- $volAdj_d$: value of water at dam d at the end of planning horizon.

$$volAdj_d = (vol_{d,P} - VOL^{final}_d) * E_d * LT^{ENGY}$$

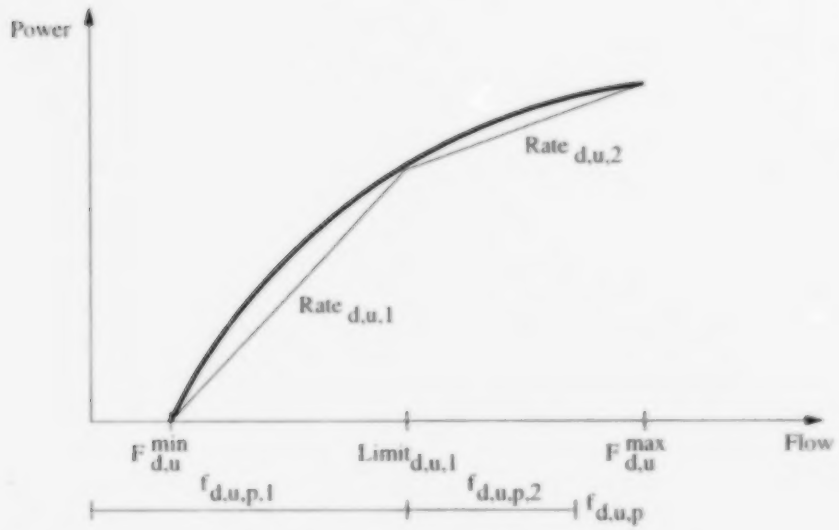

Figure 2: Example of a power generation function

3.1.7 Power generation function

For a specific unit, the power generation function $PROD_{d,u}$ is typically not linear, since the marginal increase in power tends to diminish with the flow increase. We thus approximate it through a concave piecewise linear function. An example is presented in Figure 2, using a fixed reference value for the water head. We note that there is no power until a flow that corresponds to $F^{min}_{d,u}$ is reached. The general idea is to divide the function into $I_{d,u}$ intervals with bounds $LIM_{d,u,i}$, $i = 1, ..., I_{d,u} - 1$ and $F^{max}_{d,u}$. With each interval is also associated a slope $RATE_{d,u,i}$, $i = 1, .., I_{d,u}$. We then define $I_{d,u}$ variables $f^{ENGY}_{d,u,p,i}$, $i = 1, .., I_{d,u}$, and we impose that $f^{ENGY}_{d,u,p} = \sum_{i=1,...I_{d,u}} f^{ENGY}_{d,u,p,i}$. Note that the concavity of the function is required for this model to work.

The value of $PROD_{d,u}(f^{ENGY}_{d,u,p})$ is then approximated with the following formula:

$$\sum_{i=1,...I_{d,u}} (f^{ENGY}_{d,u,p,i} * RATE_{d,u,i}) - b^{ENGY}_{d,u,p} * F^{min}_{d,u} * RATE_{d,u,1}.$$

That is, we sum the flow value associated with each interval times the rate and, if some energy is produced ($b^{ENGY}_{d,u,p} = 1$), we subtract $F^{min}_{d,u} * RATE_{d,u,1}$ because the function does not start at 0 but at $F^{min}_{d,u}$.

3.2 Stochastic model

In the stochastic model, different scenarios or realizations for the electricity and reserve prices are considered. The scenarios are organized into a tree structure where each node, at a given level, contains a particular realization of prices for the set of periods associated with that level. >From each node, the children nodes at the next level can be reached with a certain (conditional) probability. The root node corresponds to the "empty scenario", at the start of the production horizon, where no prices have yet realized. Figure 3 shows a tree of scenarios over an horizon of 24 periods, where the scenarios at the first level are associated with the set of periods {1,...,8}, those at the second level with set {9,...,16} and those at the third level with set {17,...,24}.

At each level, including the root but excluding the leaves, commitments (production decisions) are made for the set of periods associated with the next level, without knowing with certainty the electricity and reserve prices for these periods. In our example, production decisions are first taken at the root for periods

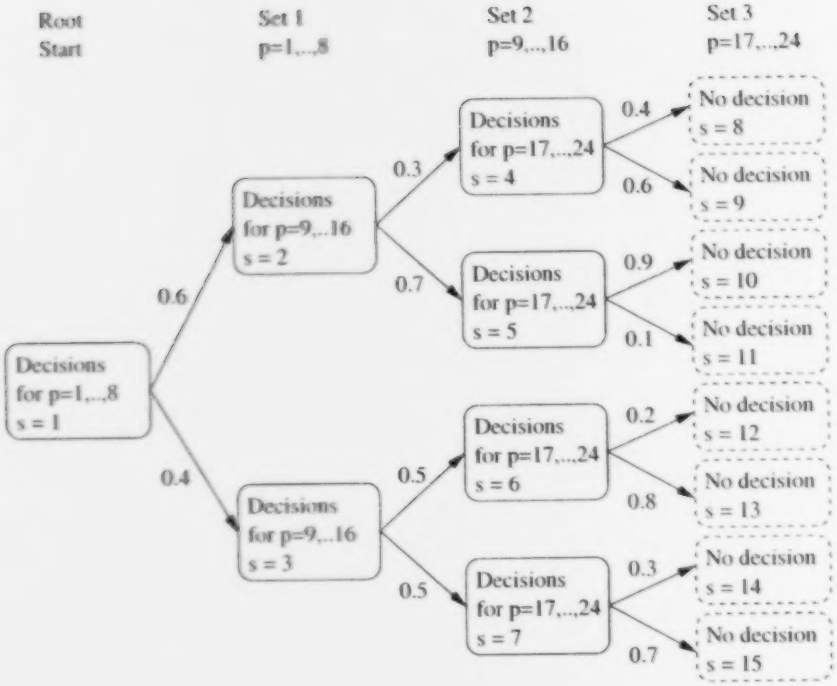

Figure 3: Tree of scenarios

1 to 8, without knowing exactly the prices for these periods. Then, based on the particular realization observed for these prices, another node is reached at the next level and new commitments are made for periods 9 to 16, etc. Although the prices are not known exactly when the production decisions are taken, the conditional probability of observing a particular realization of prices over the next set of periods, given the prices observed up to now, is assumed to be known. It is thus possible to compute the conditional expected profit associated with the production decisions for the next set of periods.

The overall expected profit is then the summation over all internal nodes in the tree of the probability of being in that node (as obtained by multiplying the conditional probabilities on the path leading from the root to the node) times the conditional expectation of the profit from that node over the next set of periods. For example, the probability of being in the node associated with scenario $s = 7$ is equal to $0.4 * 0.5 = 0.2$. Note that the leaves are left apart, given that no decision is taken there.

The shape of the tree depends on the sets of periods that are associated with each level. For example, a much deeper tree would be obtained by associating a single period with each level. Also, the branching factor in the tree corresponds to the number of different realizations of prices that are considered for each set of periods. Thus, the following parameters determine the shape of the scenario tree:

- N_b: number of branches at each level (branching factor);
- N_s: number of sets of periods (number of levels);
- p_j: last period in period set j, $j \in \{1, .., N_s\}$.

To associate prices with the nodes in the tree of scenarios, we first generate N_v vectors of prices of cardinality P, based on some underlying stochastic process (the one used in our experiments is formally introduced in Section 4). Given a branching factor N_b and a partition of the P periods into N_s sets of periods, we sort these vectors based on the price at period p_1 (last period of the first set), starting with the smallest price. We then take the price averages for periods 1 to p_1 based on the first $\lfloor N_v/N_b \rfloor$ vectors, which are those with the smallest prices, and we associate these averages and the corresponding subset of price vectors with the first child node. We repeat for the next child node by averaging over the next $\lfloor N_v/N_b \rfloor$ vectors. This is repeated until every child node of the root is done. Then, we go to the next level and repeat the whole procedure again for every node at that level: we sort the subset of vectors associated with the current node based on the price of period p_2 (last period of the second set) and associate a (sub)subset of vectors and its corresponding averages with every child. The procedure stops at the leaves of the scenario tree.

The mathematical model will now be formally introduced. It is based on the tree of scenarios, where each node is associated with a set of periods, a price scenario (realization of prices up to this set of periods) and production decisions for the next set of periods. To simplify the formulation, the model does not handle sets of periods explicitly, but rather handles each period individually.

3.2.1 Parameters

Here are the new parameters for the stochastic model (see section 3.1.1 for the other ones).

- S: number of internal nodes in the scenario tree.
- S_p : set of nodes where a decision is taken for period p, $p = 1, .., P$.
- $PARENT_{s,p,t}$: ancestor of node s, where a decision is taken for period $p - t$, $p = 1, .., P$; $s \in S_p$; $t = 0, .., p - 1$. Note that the ancestor node can be the same as node s if p and $p - t$ are in the same set of periods.
- $PROB_{p,s}$: probability to be in node s when a decision is taken for period p. It is obtained by multiplying the conditional probabilities on the path from the root to the corresponding scenario in the tree of scenarios, $p = 1, .., P$; $s \in S_p$. These probabilities are the same for all periods in the same set of periods.
- $P_{p,s}^{ENGY}$: conditional expectation of price for each MWh of energy produced at period p when a decision for period p is taken in node s ($/MWh), $p = 1, .., P$; $s \in S_p$.

- $P_{p,s}^{10S}$: conditional expectation of price for each MWh offered in "10 minutes spin reserve" when a decision for period p is taken in node s (\$/MWh), $p = 1, .., P;\ s \in S_p$.
- $P_{p,s}^{10N}$: conditional expectation of price for each MWh offered in "10 minutes non spin reserve" when a decision for period p is taken in node s (\$/MWh), $p = 1, .., P;\ s \in S_p$.

3.2.2 Decision variables

- $f_{d,p,s}^{spill} \geq 0$: flow spilled at dam d in period p when a decision for period p is taken in node s (m^3/s), $d = 1, .., D;\ p = 1, .., P;\ s \in S_p$.
- $f_{d,u,p,s}^{ENGY} \geq 0$: flow going through unit u of dam d to produce electricity in period p, when a decision for period p is taken in node s (m^3/s), $d = 1, .., D;\ u = 1, .., U_d;\ p = 1, .., P;\ s \in S_p$.
- $f_{d,u,p,i,s}^{ENGY} \geq 0$: contribution of piecewise part i, in the linear approximation of $PROD_{d,u}(f)$, to the flow going through unit u of dam d in period p to produce electricity, when a decision for period p is taken in node s (m^3/s), $d = 1, .., D;\ u = 1, .., U_d;\ p = 1, .., P;\ i = 1, .., I_{d,u};\ s \in S_p$.
- $b_{d,u,p,s}^{ENGY} \in \{0, 1\}$: 1 if electricity is produced by unit u of dam d in period p when a decision for period p is taken in node s, 0 otherwise, $d = 1, .., D;\ u = 1, .., U_d;\ p = 1, .., P;\ s \in S_p$.
- $b_{d,u,p,s}^{spin} \in \{0, 1\}$: 1 if unit u of dam d spins by supplying electricity to it in period p when a decision for period p is taken in node s, 0 otherwise, $d = 1, .., D;\ u = 1, .., U_d;\ p = 1, .., P;\ s \in S_p$.
- $mw_{d,u,p,s}^{ENGY} \geq 0$: power produced by unit u of dam d in period p when a decision for period p is taken in node s (MW), $d = 1, .., D;\ u = 1, .., U_d;\ p = 1, .., P;\ s \in S_p$.
- $mw_{d,u,p,s}^{10S} \geq 0$: power reserved for "10 minutes spin reserve" for unit u of dam d in period p when a decision for period p is taken in node s (MW), $d = 1, .., D;\ u = 1, .., U_d;\ p = 1, .., P;\ s \in S_p$.
- $mw_{d,u,p,s}^{10N} \geq 0$: power reserved for "10 minutes non spin reserve" for unit u of dam d in period p when a decision for period p is taken in node s (MW), $d = 1, .., D;\ u = 1, .., U_d;\ p = 1, .., P;\ s \in S_p$.

3.2.3 Constraints on decision variables

- (C1') $f_{d,u,p,s}^{ENGY}$ is the sum of all piecewise parts, $d = 1, .., D;\ u = 1, .., U_d;\ p = 1, .., P;\ s \in S_p$.

$$f_{d,u,p,s}^{ENGY} = \sum_{i=1,..,I_{d,u}} f_{d,u,p,i,s}^{ENGY}$$

- (C2') Bounds on $f_{d,u,p,s}^{ENGY}$, $d = 1, .., D;\ u = 1, .., U_d;\ p = 1, .., P;\ s \in S_p$.

$$0 \leq f_{d,u,p,s}^{ENGY} \leq F_{d,u}^{max}$$

- (C3') Bounds on $f_{d,u,p,i,s}^{ENGY}$, $d = 1, .., D;\ u = 1, .., U_d;\ p = 1, .., P;\ i = 1, .., I_{d,u};\ s \in S_p$.

$$\begin{aligned} i = 1: &\quad 0 \leq f_{d,u,p,i,s}^{ENGY} \leq LIM_{d,u,1} \\ i = 2, .., I_{d,u} - 1: &\quad 0 \leq f_{d,u,p,i,s}^{ENGY} \leq LIM_{d,u,i+1} - LIM_{d,u,i} \\ i = I_{d,u}: &\quad 0 \leq f_{d,u,p,i,s}^{ENGY} \leq F_{d,u}^{max} \end{aligned}$$

- (C4') Calculation of $mw_{d,u,p,s}^{ENGY} = PROD_{d,u}(f_{d,u,p,s}^{ENGY})$ through the linear approximation, $d = 1, .., D;\ u = 1, .., U_d;\ p = 1, .., P;\ s \in S_p$.

$$mw_{d,u,p,s}^{ENGY} = \sum_{i=1,..,I_{d,u}} (f_{d,u,p,i,s}^{ENGY} * RATE_{d,u,i}) - b_{d,u,p,s}^{ENGY} * F_{d,u}^{min} * RATE_{d,u,1}$$

- (C5') $f_{d,u,p,s}^{ENGY}$ equals 0 if $b_{d,u,p,s}^{ENGY}$ equals 0 and is between a minimum and a maximum value otherwise, $d = 1, .., D;\ u = 1, .., U_d;\ p = 1, .., P;\ s \in S_p$.

$$F_{d,u}^{min} \cdot b_{d,u,p,s}^{ENGY} \leq f_{d,u,p,s}^{ENGY} \leq F_{d,u}^{max} \cdot b_{d,u,p,s}^{ENGY}$$

- (C6') Spill flow of dam d in period p is between a minimum and a maximum value, $d = 1, .., D;\ u = 1, .., U_d;\ p = 1, .., P;\ s \in S_p$.

$$F_d^{spill_{min}} \leq f_{d,p,s}^{spill} \leq F_d^{spill_{max}}$$

- (C7') Maximum power of unit u of dam d is used in period p, $d = 1,..,D;\ p = 1,..,P;\ s \in S_p$.

$$mw^{ENGY}_{d,u,p,s} + mw^{10S}_{d,u,p,s} + mw^{10N}_{d,u,p,s} = MW^{max}_{d,u}$$

- (C8') Power for "10 minutes spin reserve" can only be reserved if unit u of dam d spins or produces electricity, $d = 1,..,D;\ u = 1,..,U_d;\ p = 1,..,P;\ s \in S_p$.

$$mw^{10S}_{d,u,p,s} \leq MW^{max}_{d,u} \cdot (b^{ENGY}_{d,u,p,s} + b^{spin}_{d,u,p,s})$$

- (C9') Unit u of dam d cannot produce electricity and spin at the same time, $d = 1,..,D;\ u = 1,..,U_d;\ p = 1,..,P;\ s \in S_p$.

$$b^{ENGY}_{d,u,p,s} + b^{spin}_{d,u,p,s} \leq 1$$

3.2.4 Intermediate variables

- $inflow_{d,p,s}$: flow that enters the reservoir of dam d in period p when a decision for period p is taken in node s (m^3/s), $d = 1,..,D;\ p = 1,..,P;\ s \in S_p$.

$$inflow_{d,p,s} = NAT_IFd,p + \sum_{\substack{z=1,..,D \\ t=0,..,N}} LK_{z,d} * outflow_{z,p-t,PARENT_{z,p,t}} * LK_FRAC(z,d,t)$$

- $outflow_{d,p,s}$: flow that leaves the reservoir of dam d in period p, when a decision for period p is taken in node s (m^3/s), $d = 1,..,D;\ p = 1,..,P;\ s \in S_p$. It is also assumed that a number of "historical" outflow values are available for periods $-(N-1),..,0$.

$$outflow_{d,p,s} = f^{spill}_{d,p,s} + \sum_{u=1,..,U_d} f^{ENGY}_{d,u,p,s}$$

- $vol_{d,p,s}$: volume of dam d at the end of period p, when a decision for period p is taken in node s (m^3), $d = 1,..,D;\ p = 1,..,P;\ s \in S_p$.

$$p = 1: \quad vol_{d,p,s} = VOL^{init}_d + (inflow_{d,p,s} - outflow_{d,p,s}) * DUR_p * 3600$$
$$p = 2,..,P: \quad vol_{d,p,s} = vol_{d,p-1,Parent_{z,p,1}} + (inflow_{d,p,s} - outflow_{d,p,s}) * DUR_p * 3600$$

3.2.5 Constraints on intermediate variables

- (C10') Volume of dam d at the end of period p is between a minimum and maximum value, when a decision for period p is taken in node s, $d = 1,..,D;\ p = 1,..,P;\ s \in S_p$.

$$VOL^{min}_d \leq vol_{d,p,s} \leq VOL^{max}_d$$

- (C11') Outflow of dam d in period p is between a minimum and maximum value, when a decision for period p is taken in node s, $d = 1,..,D;\ p = 1,..,P;\ s \in S_p$.

$$F^{out_{min}}_d \leq outflow_{d,p,s} \leq F^{out_{max}}_d$$

3.2.6 Objective

- Objective function (\$)

$$Maximize \sum_{\substack{d\in\{1,..,D\} \\ u\in\{1,..,U_d\} \\ p\in\{1,..,P\} \\ s\in S_p}} Prob_{p,s} \cdot (revenue_{d,u,p,s} - c^{spin}_{d,u,p,s} - c^{start}_{d,u,p,s}) + \sum_{\substack{d\in\{1,..,D\} \\ s\in S_P}} Prob_{P,s} \cdot volAdj_{d,s}$$

- $revenue_{d,u,p,s}$: conditional expectation of revenue from selling electricity and reserve for unit u of dam d in period p, when a decision for period p is taken in node s (\$), $d = 1,..,D;\ u = 1,..,U_d;\ p = 1,..,P;\ s \in S_p$.

$$revenue_{d,u,p,s} = (mw^{ENGY}_{d,u,p,s} \cdot P^{ENGY}_{p,s} + mw^{10S}_{d,u,p,s} \cdot P^{10S}_{p,s} + mw^{10N}_{d,u,p,s} \cdot P^{10N}_{p,s}) \cdot DUR_p$$

- $c^{spin}_{d,u,p,s}$: conditional expectation of cost for providing electricity to unit u of dam d to make it spin in period p, when a decision for period p is taken in node s (\$), $d = 1,..,D;\ u = 1,..,U_d;\ p = 1,..,P;\ s \in S_p$.

$$c^{spin}_{d,u,p,s} = b^{spin}_{d,u,p,s} \cdot MW^{spin}_{d,u} \cdot P^{ENGY}_{p,s} \cdot DUR_p$$

- $c^{start}_{d,u,p,s}$: starting cost of unit u of dam d in period p, when a decision for period p is taken in node s (\$), $d = 1,..,D;\ u = 1,..,U_d;\ p = 1,..,P;\ s \in S_p$.

$$p = 1: \quad c^{start}_{d,u,p,s} \geq C_START_{d,u} \cdot b^{ENGY}_{d,u,p,s},$$
$$p = 2,..,P: \quad c^{start}_{d,u,p,s} \geq C_START_{d,u} \cdot (b^{ENGY}_{d,u,p,s} - b^{ENGY}_{d,u,p-1,PARENT_{s,p,1}}),$$
$$c^{start}_{d,u,p,s} \geq 0$$

- $volAdj_{d,s}$: value of water at dam d at the end of planning horizon when a decision for period P is taken in node s (\$), $d = 1,..,D;\ s \in S_P$.

$$volAdj_{d,s} = (vol_{d,P,s} - VOL^{final}_d) * E_d * LT^{ENGY}$$

4 Numerical results

This section summarizes the results of three different types of experiments. First, we examine the impact of the horizon length on both the deterministic and stochastic models. Then, we compare the two models and demonstrate the superiority of the stochastic model with regard to solution quality. Finally, the impact of price volatility is examined.

Average electricity ($\overline{P}^{ENGY}_p$) and reserve prices ($\overline{P}^{10S}_p, \overline{P}^{10N}_p$) are used at each period $p \in P$. The latter come from historical data available on a market web site. In the deterministic case, these values are fed directly into the model for optimization purposes, that is, $P^{ENGY}_p = \overline{P}^{ENGY}_p$, $P^{10S}_p = \overline{P}^{10S}_p$ and $P^{10N}_p = \overline{P}^{10N}_p$.

For the stochastic model, the branching factor N_b at each level and the number of sets of periods or levels N_s are first specified. Then, the value of the p_j's are chosen to get (as much as possible) the same number of periods at each level. That is:

$$p_j = \lfloor P * j/N_s \rfloor, \quad j = 1,..,N_s - 1,$$
$$p_j = P, \quad j = N_s.$$

The stochastic process for generating the electricity prices in our experiments, denoted ϕ^{ENGY}_p, $p = 1,...,P$, is:

- $p = 1$,

$$\phi^{ENGY}_p = \begin{cases} \pi_p * a_1 & if\ \epsilon_p \leq Q, \\ \pi_p * (2 - a_1) & if\ Q < \epsilon_p \leq 2Q, \\ \pi_p & if\ \epsilon_p > 2Q \end{cases}$$

- $p = 2,..,\lfloor P/4 \rfloor$,

$$\phi^{ENGY}_p = \begin{cases} \phi^{ENGY}_{p-1} * \pi_p * a_1 & if\ \epsilon_p \leq Q, \\ \phi^{ENGY}_{p-1} * \pi_p * (2 - a_1) & if\ Q < \epsilon_p \leq 2Q, \\ \phi^{ENGY}_{p-1} * \pi_p & if\ \epsilon_p > 2Q \end{cases}$$

- $p = \lfloor P/4 \rfloor + 1,..,\lfloor 2P/4 \rfloor$,

$$\phi^{ENGY}_p = \begin{cases} \phi^{ENGY}_{p-1} * \pi_p * a_2 & if\ \epsilon_p \leq Q, \\ \phi^{ENGY}_{p-1} * \pi_p * (2 - a_2) & if\ Q < \epsilon_p \leq 2Q, \\ \phi^{ENGY}_{p-1} * \pi_p & if\ \epsilon_p > 2Q \end{cases}$$

- $p = \lfloor 2P/4 \rfloor + 1, ..., \lfloor 3P/4 \rfloor$,

$$\phi_p^{ENGY} = \begin{cases} \phi_{p-1}^{ENGY} * \pi_p * a_3 & if\ \epsilon_p \leq Q, \\ \phi_{p-1}^{ENGY} * \pi_p * (2 - a_3) & if\ Q < \epsilon_p \leq 2Q, \\ \phi_{p-1}^{ENGY} * \pi_p & if\ \epsilon_p > 2Q \end{cases}$$

- $p = \lfloor 3P/4 \rfloor + 1, ..., P$,

$$\phi_p^{ENGY} = \phi_{p-1}^{ENGY} * \pi_p + \frac{\overline{P}_p^{ENGY} - \phi_{p-1}^{ENGY} * \pi_p}{1 + \alpha},$$

where a_1, a_2 and a_3 are the "volatility" parameters, α is a parameter that controls the convergence rate toward the average, $2Q$ is the probability to deviate from the average price (probability Q to be over the average; probability Q to be under the average), ϵ_p, $p = 1, ..., \lfloor 3P/4 \rfloor$, are independent uniform random variables in the interval [0,1] and

$$\pi_p = \overline{P}_p^{ENGY} \quad p = 1$$
$$\pi_p = \overline{P}_p^{ENGY} / \overline{P}_{p-1}^{ENGY}, \quad p = 2, ..., P.$$

With this definition of ϕ_p^{ENGY} it is easy to show that $E[\phi_p^{ENGY}] = \overline{P}_p^{ENGY}$, $p = 1, ..., P$. Furthermore, the conditional probability to observe the prices associated with a child node (for the next set of periods) from any given node is $1/N_b$. For $p = 1, ..., \lfloor 3P/4 \rfloor$, the prices deviate from the average through a multiplicative process based on different volatility parameters. Then, for $p = \lfloor 3P/4 \rfloor + 1, ..., P$ the process becomes deterministic and converges toward the average at rate α (the lower is α, the faster is the convergence). Clearly, the correlation between electricity prices in consecutive periods is high, but this is exactly what is observed in real markets. This is the same for the "return toward the average" which is typically observed at the end of the day. For the two reserve types "10 minutes spin" and "10 minutes non spin", deterministic prices are used, that is, $\phi_p^{10S} = \overline{P}_p^{10S}$, $\phi_p^{10N} = \overline{P}_p^{10N}$, $p = 1, ..., P$.

To associate average prices with the nodes in the tree of scenarios, the number of price vectors N_v was set to $100 * {N_b}^{N_s}$ (see subsection 3.2). This ensures that the averages computed at the leaves are based on a representative subset of 100 price vectors. For example, the values $N_b = 2$ and $N_s = 2$ correspond to a tree of depth 2, where each internal node has two children. Consequently, from $100 * 2^2 = 400$ price vectors, two subsets of 200 price vectors are associated with the two children node of the root and four (sub)subsets of 100 price vectors are associated with the four leaves.

Both the deterministic and stochastic models were implemented in CPLEX. The maximal size of the CPLEX search tree was limited to 1024 MB. All numerical tests were executed on a 3 GHz Pentium 4, with 1 GB of memory.

4.1 Increasing the number of periods

This experiment analyzes the impact of increasing the number of planning periods on computation times. Table 3 shows the execution time and gap for different numbers of periods when solving the deterministic and stochastic models. In the latter case, the parameters were set to: $N_b = 3$, $N_s = 4$, $a_1 = 1.1$, $a_2 = 1.2$, $a_3 = 1.15$, $\alpha = 2$, $Q = 0.3$. Note that a time limit of 7200 seconds, or 2 hours, was enforced in all experiments. In the Table, the (relative) gap shown corresponds to the difference between the upper bound obtained by relaxing the integrality constraints and the best integer solution found, divided by the upper bound. A gap of zero indicates an optimal solution.

We can see that the deterministic model can find optimal solution in less than a minute for up to 384 periods. It is even possible to obtain an optimal solution to a problem with 768 periods in a little bit more than 8 minutes. The computational burden of the stochastic model is much heavier and optimal solutions are found for problems with up to 48 periods only. However, this is enough for practical purposes because the planning horizon in our application extends over 24 periods (hours). It should be noted that for $P = 96$,

	Deterministic		Stochastic	
P	Time (s)	Rel. gap (%)	Time (s)	Rel. gap (%)
12	0.06	0	0.89	0
24	0.11	0	119.9	0
48	0.29	0	4921	0
96	3.23	0	5136	0.00221
192	12.4	0	7200	0.15709
384	32.8	0	7200	0.03997
768	490	0	-	-
1536	7200	0.0002	-	-
3072	7200	0.00003	-	-

Table 3: Impact of increasing the number of periods

the preset memory limit was reached by the search tree. Hence, there is a gap even if the total computation time is less than 7200 seconds. Note also that no results are reported for the stochastic model for values of P greater than or equal to 768, due to memory limitations.

4.2 Branching factor and number of levels

This experiment studies the impact of the branching factor and number of levels in the tree of scenarios on solution quality. The latter was evaluated through a Monte Carlo approach. That is, we first generated optimal production plans with CPLEX for trees of scenarios of different shapes by using different values for parameters N_b and N_s. Then, each production plan obtained (i.e., production decisions for each period) was evaluated on 100,000 new price vectors produced with our stochastic price generation scheme. We recall that an average price is stored at each node in the tree of scenarios for each period in the corresponding set of periods, based on a subset of price vectors (see subsection 3.2). Considering only the last period in the period set, which is the most important with regard to the prices that will be observed in the next set of periods, the maximum price value is taken over the subset of price vectors at each node. The latter values are then used to follow a path of production decisions in the tree of scenarios. Let assume, for example, that nodes b, c and d (from smallest to largest prices) are the three children nodes of a given node a and are associated with period set $j \in N_s$. We denote the maximum price value of node n by P_n^{max} and the price of period p_j in some newly generated price vector by $P_{p_j}^{new}$. Then, from node a, we go to:

- node b if $P_{p_j}^{new} \leq P_b^{max}$
- node c if $P_b^{max} < P_{p_j}^{new} \leq P_c^{max}$
- node d otherwise.

Table 4 reports the results for different values of N_b and N_s. The numbers in the table correspond to the ratio of the solution obtained with a given set of parameters to the best solution found with $N_b = 5$ and $N_s = 5$, which is the most refined model. In these experiments, the parameters were set to: $P = 24$, $a_1 = 1.1$, $a_2 = 1.2$, $a_3 = 1.15$, $\alpha = 2$, $Q = 0.3$. The computation was stopped as soon as the relative gap dropped below 0.01%. The time limit was set to 7200 seconds (2 hours), but the relative gap dropped below 0.01% in less than one hour even on the largest instances.

The results obtained with $N_b = 1$ or $N_s = 1$ correspond to the results that would have been obtained with the deterministic model. That is why these results are so similar. The small variation of 0.05% between the minimum and maximum values can be attributed to random noise. What is more important is the significant gap observed between the performance of the deterministic and stochastic models. Furthermore, this gap widens when the stochastic model is more refined (i.e., when the values of parameters N_b and N_s increase). In fact, the improvement over the deterministic model is almost 8% when N_b=5 and N_s=5.

$N_b \setminus N_s$	1	2	3	4	5
1	.9208	.9207	.9211	.9212	.9208
2	.9210	.9685	.9698	.9806	.9901
3	.9209	.9763	.9806	.9877	.9955
4	.9212	.9770	.9808	.9913	.9988
5	.9206	.9779	.9818	.9935	1

Table 4: Ratio to the best solution found (with N_b=5 and N_s=5)

$a_1 = a_2 = a_3$	ratio
1.00	.717
1.05	.727
1.10	.748
1.15	.775
1.20	.804
1.25	.836
1.30	.871
1.35	.906
1.40	.936
1.45	.969
1.50	1

Table 5: Impact of price volatility

4.3 Price volatility

The last experiment evaluates the impact of price volatility (or variance) on the performance of the stochastic model. The results are shown in Table 5. Each entry in this table is the ratio of the objective value to the value found with the largest price volatility tested (i.e., $a_1 = a_2 = a_3 = 1.5$).

In these experiments, the parameters were set to: $P = 24$, $N_b = 3$, $N_s = 4$, $\alpha = 2$, $Q = 0.3$. A stopping criterion based on a relative gap of 0.01% was also used here. With this stopping criterion, all problems were solved in less than 30 seconds of computation time. The results clearly show that the improvement of the stochastic model over the deterministic one ($a_1 = a_2 = a_3 = 1$) increases with price volatility. In fact, a steady improvement is observed when the values of the volatility parameters increase from 1 to 1.5.

5 Conclusion

This paper has introduced two mathematical models to maximize the profits obtained from selling hydro-electricity in a deregulated energy market. The first model is based on deterministic prices while the second one integrates stochastic prices by considering a number of different price realizations that are organized into a tree structure. The numerical results shows that the stochastic model is superior to the deterministic one with regard to solution quality. Also, the performance of the stochastic model improves when prices are highly volatile.

Future developments will now be aimed at integrating the bidding process observed in deregulated markets within the stochastic model. By considering these bids, price volatility could be better exploited. Indeed, with a clever bidding strategy, electricity can be sold when prices are unexpectedly high, even if that was not planned in the original plan. The same thing is true for prices that happen to be unexpectedly low. Thus, additional profits can be expected from this integration.

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